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LETTER TO THE EDITOR

Stochastic traffic model with random deceleration probabilities: queueing and power-law gap distribution

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Abstract. We extend the Nagel–Schreckenberg stochastic cellular automata model for single-lane vehicular traffic to incorporate quenched random deceleration probabilities. We show, by computer simulations, that at low densities this model displays queueing of cars with a power-law probability distribution of gaps between the cars while at high densities the behaviour of the model is similar to the jammed phase of the standard Nagel–Schreckenberg model. The approach to the steady state is characterized by the same critical exponents as for the coarsening process in the simple exclusion processes with random rates, recently investigated independently by Krug and Ferrari, and Evans. The numerical values of the exponents for gap distributions are in agreement with the analytical conjecture of Krug and Ferrari, which implies that the models belong to the same universality class.

During the last few years traffic problems have been investigated intensively using particle-hopping models [1]. One of these models, introduced by Nagel and Schreckenberg (NS) [2] and formulated in the language of cellular automata, has been quite successful in reproducing the qualitative features of real traffic [3]. This model is characterized by two dynamical phases—a low-density laminar phase and a high-density jammed phase. Recently, Benjamini, Ferrari and Landim (BFL) [4] introduced a class of exactly solvable particle-hopping models which generalize the asymmetric simple exclusion process (ASEP) [5]. Krug and Ferrari [6] and Evans [7] investigated independently a simplified version of the BFL model and showed that the jammed phase appears in this model at low densities of cars. The aim of this letter is to propose a variant of the NS model, which also has jamming at low densities and to compare the features of the traffic flow in this new variant to the corresponding properties of the BFL model, considered by Krug and Ferrari, and Evans (further referred to as the 'BFL model').

We recall the original formulations of the ASEP and the NS model. In both, single-lane traffic on a ring of length L is modelled as a lattice of L sites with periodic boundary conditions. Each of the L sites can either be empty or occupied by one vehicle.

In the ASEP one particle is picked up at random and moved forward by one site if the new site is empty. In this random sequential update it is the random picking that introduces

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stochasticity (noise) into the model. Thus, the maximum possible speed of a particle in the ASEP is $v_{\max} = 1$. In contrast, in the NS model, the speed v of each vehicle can take one of the $v_{\max} + 1$ allowed integer values $v = 0, 1, \ldots, v_{\max}$. At each discrete time step $t \to t + 1$, the arrangement of N cars is updated according to the following rules.

(1) *Acceleration.* If the speed v of a vehicle is lower than $v_{\max}$, the speed is advanced by one ($v = v + 1$).

(2) *Slowing down (due to other cars).* If the distance d to the next car ahead is not larger than $v (d \leqslant v)$, the speed is reduced to $d - 1 (v = d - 1)$.

(3) *Randomization.* With probability p, the speed of a vehicle (if greater than zero) is decreased by one ($v = v - 1$); we shall refer to p as the *deceleration* probability.

(4) *Car motion.* Each vehicle is advanced v sites.

Very recently, Krug and Ferrari [6] and Evans [7] have studied ASEP where the jump rate of each individual particle is chosen initially from a suitable distribution of random numbers. Suppose $u_i = x_{i+1} - x_i - 1$ denotes the number of vacant sites in front of the ith particle. Interpreting u_i as a particle occupation number in a state i, the occurrence of a Bose–Einstein-like macroscopic condensation in one of the states was demonstrated. In this 'Bose-condensed state' a finite fraction of the empty sites are condensed in front of the slowest particle.

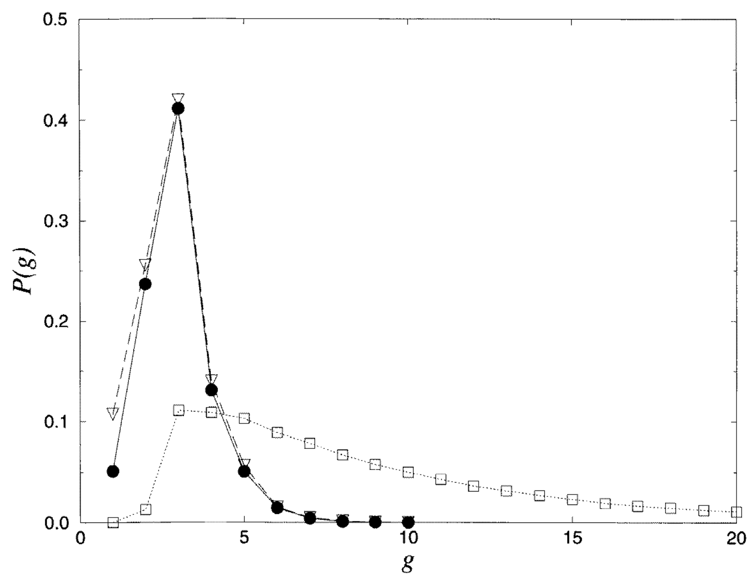

Figure 1. Gap distribution for the model with two deceleration parameters (circles) at a density of 0.1 ($p_1 = 0.5$, $p_2 = 0.1$) and the same distribution for the standard NS model at a density of 0.35 and 0.1 ($p = 0.1$) (triangles and squares correspondingly).

Now we generalize the NS model and present our results. Let us first consider the NS model with two different values of the deceleration probability p. Suppose a randomly chosen fraction f of the drivers have the deceleration probability p_1 whereas the remaining drivers have the deceleration probability p_2, where $p_1 > p_2$. Since the deceleration probability of each driver is fixed once for all in the beginning and is not changed during the time evolution of the system, the disorder in the braking probability is quenched. More 'careful' drivers in our model are characterized by a higher value of the

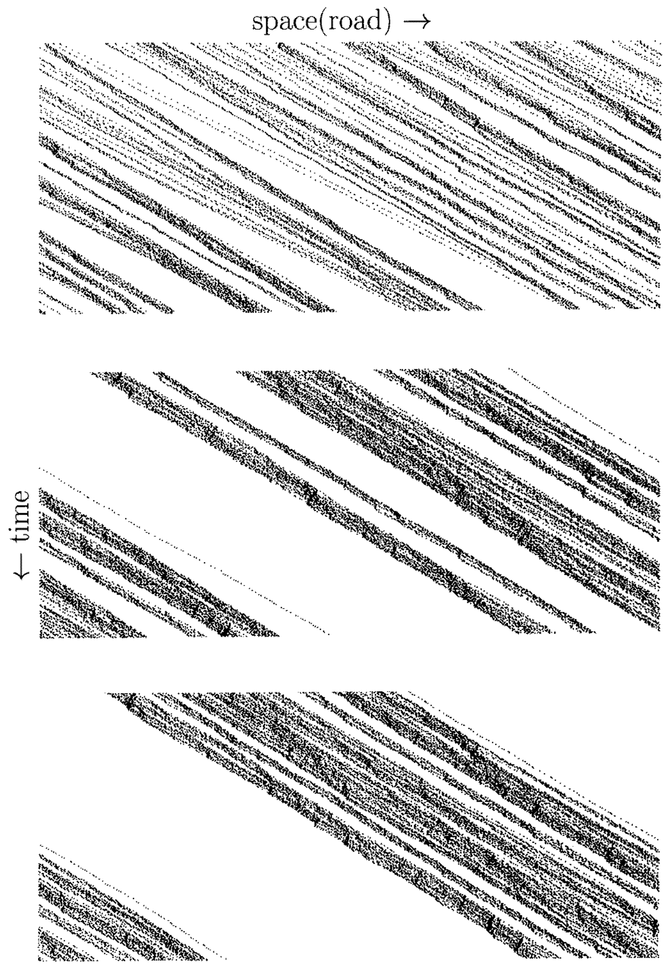

Figure 2. Space–time plot of the random NS model with $v_{\max} = 2$ and deceleration probabilities distributed by the density P_1 (see text for formula), the car density is 0.1, length of lane $L = 1000$. Dots are cars moving to the right. The vertical direction (down) is increasing time. The three time intervals shown are separated by approximately 15 000 time steps. One long queue appears after about 30 000 time steps.

deceleration probability p; these drivers tend to brake more often and accelerate slowly. On the other hand, those 'careless' drivers who accelerate quickly correspond to a smaller value of p. Sometimes such 'careless' drivers are also referred to as 'aggressive'. The behaviour of the model with a small number of 'careful' drivers is quite predictable: very soon there appear several queues of cars, each led by a 'careful' driver.

In the BFL model [6] the queueing phase at low density occurs when the 'slow' cars, even when a minority, hinder the movement of the fast ones behind. The 'slow' cars in the BFL model are the analogues of the 'careful' drivers in our model.

The NS model with $v_{\max} = 1$ and the BFL model differ only by the type of update (parallel or random sequential [3], respectively). However, for $v_{\max} > 1$ the qualitative behaviour of the two models may be different. Our generalization of the NS model combines the acceleration and slowing down of NS with the quenched random deceleration

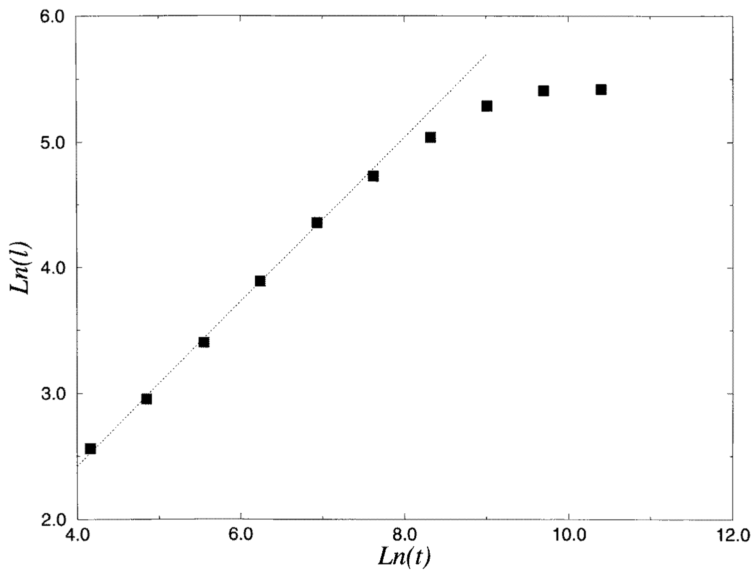

Figure 3. Log–log plot of typical queue length against time for the model presented in figure 2. The dotted curve has a slope of 0.66.

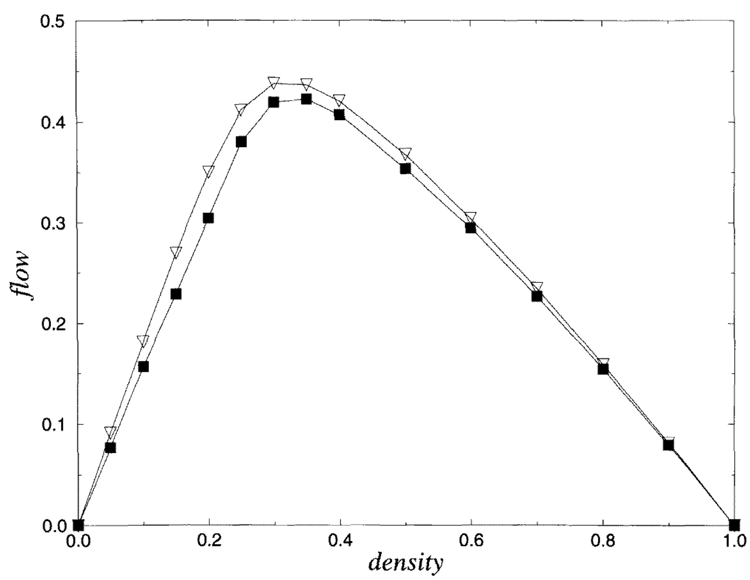

Figure 4. Fundamental diagram for the random NS model ($v_{\max} = 2$) with deceleration probability distribution P_1 (squares) and for the standard NS model with deceleration probability $p = \langle p \rangle_{P_1} = 1/6$ (triangles).

probabilities in the spirit of BFL. In this letter, we compare the main features of random NS models with $v_{\max} = 1$ and $v_{\max} = 2$ with the corresponding properties of the BFL model.

We considered the two-parameter model for the length of freeway $L = 1000$, total density of cars $\rho = 0.1$ and the fraction of slow drivers $f = 0.1$. The probability distribution

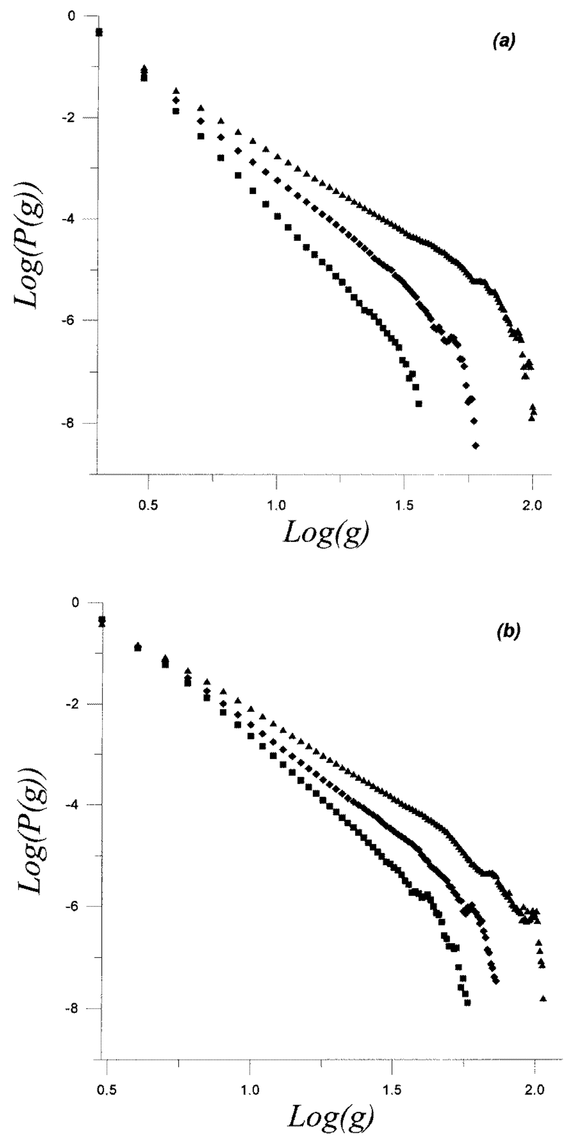

Figure 5. Log–log plots of gap distributions at critical densities for the random NS model ((*a*) for $v_{\max} = 1$, (*b*) for $v_{\max} = 2$) with deceleration distributions P_n ($n = 1$, triangles; $n = 2$, diamonds; $n = 3$, squares).

of the gaps in front of the cars in this model is characterized by the presence of the peak at the point $v_{\max} + 1$; it is qualitatively different from the corresponding distribution in the NS model at the same density 0.1 (laminar phase), but is quite close to the gap distribution of the jammed phase of the NS model at a higher density of 0.35 (figure 1).

This example shows that even a small perturbation of the stochastic parameter p in the NS model leads to locally jammed situations even at low densities.

Next, we consider a more realistic situation where the deceleration probabilities p of the drivers are chosen from a common random distribution with a decreasing density function

$P(p)$. We take for our simulations

$$P_n(p) = 2^n(n+1)(\tfrac{1}{2} - p)^n \qquad 0 \leqslant p \leqslant \tfrac{1}{2} \qquad n = 1, 2, 3.$$

The behaviour of this model is more complicated than the example considered above. In the model with random deceleration probabilities after a prolonged time interval the cars form one long queue (figure 2). We estimated the time of such a queue coarsening by measuring the two-point correlation function

$$c(\Delta x, t) = \langle \nu(x,t)\nu(x+\Delta x, t)\rangle_x - \rho^2$$

where the function $\nu = 1$ if there is a car at the point x at time t, 0 otherwise; $\langle \nu \rangle = \rho$, ρ is the car density. The first zero l of the function c corresponds to a typical queue length at time t. The simulations for both $v_{\max} = 1$ and $v_{\max} = 2$ random NS models show that $l \sim t^{1/z}$, $1/z \simeq 0.65$ for $n = 1$, 0.73 for $n = 2$ and 0.78 for $n = 3$. One of the cases ($v_{\max} = 2, n = 1$) is represented in figure 3. These estimations are in close agreement with the analytical expression for the coarsening exponent of the BFL model

$$z = \frac{n+2}{n+1}$$

obtained by Krug and Ferrari [6] from extremal statistics estimates. Using the same arguments for the traffic model with random velocities, Ben-Naim *et al* [8] derived the similar formula for the cluster growing exponent. These arguments can also be applied to our model, so it is plausible that the formula (2) is valid for the random version of the NS model.

We note that the fundamental diagram for our model and the one for the standard NS model have quite similar form (figure 4), so the qualitative distinction between the models should be formulated in other terms. The main feature of the model with random deceleration probabilities is that the system self-organizes into a stable queueing phase at low densities and has the power-law decay of its gap distribution for some critical density (figure 5). For high densities the cars are suppressed by the finite length of the lane and have no more free space for self-organization.

The similar phenomena of 'bunching of cars' was pointed out by Nagatani [9] for the NS model with $v_{\max} = 1$ and the uniform distribution of deceleration probabilities p. However, the gap distributions in that case do not possess power-law asymptotics.

Finally, we estimated numerically the critical densities ρ_c and the critical exponents α of the gap distributions $P(g) \sim g^{-\alpha}$ for our model with $v_{\max} = 1$ and $v_{\max} = 2$ for different distributions of decelerations P_n. The results are represented in table 1.

Table 1. The estimations of critical exponents α of gap distributions at critical densities ρ_c for the random NS model with different v_{max} and different probability distributions P_n of the deceleration parameter (see text).

$v_{\max}$	n	ρ_c	α
1	1	0.48	3.0
1	2	0.55	3.9
1	3	0.59	5.2
2	1	0.26	3.4
2	2	0.30	4.2
2	3	0.32	5.0

The distinction of our models from the BFL model is that for different n the transition occurs at different values of densities. The estimations of α, however, are quite close to the formula $\alpha = n + 2$ for the corresponding exponents in the BFL model, obtained by Krug and Ferrari [6]. So it is quite plausible that the random NS model and the BFL model with random rates are in the same universality classes.

In both the models of Krug and Ferrari [6] and of Evans [7] as well as in our model the quenched randomness are the characteristics of the particles (or, vehicles) while the lattice representing the highway lane is completely non-random. This should be contrasted with the traffic model considered by Csahok and Vicsek [10], where quenched random 'permeability' is associated with each lattice site, rather than randomizing p.

To conclude, we have investigated numerically the general dynamical phases of car motion in the NS model with quenched random deceleration probabilities. For a high density of cars the behaviour of the system is similar to the jammed phase of the standard NS model, for low densities the system displays the queueing of cars with power-law distribution of gaps between them at some critical point.

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